Journal of Thermophysics and Heat Transfer
Vol. 21, No. 2, April–June 2007

Classical Plane Couette Flow with Viscous Dissipation and Variable Fluid Properties

Asterios Pantokratoras*
Democritus University of Thrace, 671 00 Xanthi, Greece

DOI: 10.2514/1.23496

This study investigates the effect of variable viscosity and variable thermal conductivity on the classical plane Couette flow with viscous dissipation. The investigation concerns air, engine oil, and water, taking into account the variation of these quantities with temperature. The results are obtained with the direct numerical solution of the governing equations and cover large temperature differences. Velocity and temperature profiles are presented for equal or unequal plate temperatures in comparison with analytical solutions that are valid for constant properties. It is found that dynamic viscosity plays an important role in the results for all three fluids, whereas the influence of thermal conductivity is more important in air. In many cases, velocity and temperature profiles depart significantly from those that correspond to constant properties. For equal plate temperatures, the difference between the variable properties' Nusselt numbers and those corresponding to constant properties increases as the Brinkman number increases. For unequal plate temperatures, there is no clear trend, due to complicated interaction between the u, T, μ, and k. For water, the difference between the variable properties' Nusselt numbers and those corresponding to constant properties is small, because water viscosity and thermal conductivity are weak functions of temperature.

I. Introduction

THE first viscous fluid flow treated in the classical book by White [1] is the steady flow between a fixed and a moving plate (Couette flow) and this is discussed in almost all fluid mechanics books, because this flow is the simplest in fluid mechanics. This flow is called Couette flow in honor of Maurice Couette [2], who performed experiments on the flow between a fixed and a moving concentric cylinder (White). It is known in physics that when a body moves along another body, heat is produced, due to friction. The same happens when a fluid flows: heat is produced, due to friction between the fluid particles. This heat is usually called viscous dissipation and is significant when fluid viscosity is large or fluid velocity is large or both. A variety of expressions are used in the literature for this quantity, such as viscous dissipation, viscous heating, viscous work, shear-stress heating, and frictional heating. It is remarkable that in the Couette flow treated by White, the viscous dissipation has been taken into account. However, viscosity and thermal conductivity have been assumed constant.

The investigation of viscous dissipation has been extended to many fields of fluid mechanics and heat transfer. Hestroni et al. [3] presented a review of liquid and gas flows with viscous dissipation in microchannels. Attention was paid to comparison between predictions of theory and experimental data obtained during the last decade. Probably the most recent work on boundary-layer flow with viscous dissipation is that of Aydin and Kaya [4], who treated the mixed convection problem along a vertical flat plate of a viscous dissipating fluid. Barletta et al. [5] investigated the classical problem of the fully developed mixed convection flow with viscous dissipation in a vertical channel bounded by isothermal walls. In porous media, the problem of viscous dissipation modeling is still open and controversial, as was shown by Nield et al. [6].

Viscous dissipation is important both from scientific and practical points of view. Laminar flows have a fatal weakness of poor resistance to high Reynolds numbers. Therefore, these flows pass to a transitional, unstable stage and change to turbulent when the Reynolds number exceeds some critical value. In recent years, extensive research has been conducted on stability of fluid flows and among them is the plane Couette flow, due to its simplicity. However, in the stability of Couette flows, viscous dissipation and the variation of fluid properties with temperature are ignored. Duck et al. [7] were probably the first who conducted a linear stability analysis of Couette air flow that took into account the frictional heating and temperature-dependent viscosity. Hu and Zhong [8] extended the work of Duck et al. to higher Mach numbers. In both works, the lower plate was adiabatic and thermal conductivity was constant. From a scientific point of view, the plane Couette flow with viscous dissipation and variable fluid properties is a very good basic flow for the application of stability analysis.

Except that there are numerous physical phenomena for which viscous dissipation is important, as happens in glaciology, in interaction between the crust and the mantle, in separation of the oceanic crust from the lithosphere, and in rock avalanches. A typical coefficient of friction for one piece of rock sliding past another might be about 0.55. In large rock avalanches, however, the coefficient of friction necessary to explain the travel distance can be as low as 0.1. An explanation of this phenomenon can be given by assuming that rock avalanches glide atop pockets of air trapped between rock pieces. In civil engineering structures, fluid dampers are used to suppress earthquake- and wind-induced vibrations. In these devices, energy is converted into heat through viscous dissipation (Makris [9]). High-pressure homogenization is a novel method for milk treatment in which significant heating of the milk is caused by frictional heating (Datta et al. [10]). In tribology, the oil temperature rise becomes considerable, due to friction (Schlichting [11], p. 276), and the same result happens in aeronautics, in which the air temperature rises up to 2000°C for Mach number 6 (Schlichting, p. 311).

The plane Couette flow with viscous dissipation has been studied in the past, but usually with constant viscosity and thermal conductivity. The literature on this problem concerning fluids with variable physical properties is scarce. In the present work, we treat, in a systematic way, the problem of plane Couette flow for air, engine oil, and water, taking into account the viscous dissipation and the variation of both viscosity and thermal conductivity with temperature. The temperature range is 150–3000 K for air, 273–430 K for engine oil, and 273–373 K for water.

II. Mathematical Model

The momentum equation for the fully developed plane Couette flow with variable viscosity is

Received 27 February 2006; revision received 2 August 2006; accepted for publication 23 August 2006.

*School of Engineering; apantokr@civil.duth.gr.

$$\frac{\partial}{\partial y}\left(\mu\frac{\partial u}{\partial y}\right)=0 \tag{1}$$

where x is the horizontal coordinate, y is the vertical coordinate, u is the velocity along the plates, and μ is the fluid dynamic viscosity. The energy equation in general form is

$$\rho\left(u\frac{\partial h}{\partial x}+v\frac{\partial h}{\partial y}\right)=\frac{\partial}{\partial y}\left(k\frac{\partial T}{\partial y}\right)+\mu\left(\frac{\partial u}{\partial y}\right)^2 \tag{2}$$

where ρ is the fluid density, h is the fluid enthalpy, T is the fluid temperature, and k is the fluid thermal conductivity. The temperature formulation of the energy equation is (Bejan [12], p. 14)

$$\rho c_p\left(u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}\right)=\frac{\partial}{\partial y}\left(k\frac{\partial T}{\partial y}\right)+\mu\left(\frac{\partial u}{\partial y}\right)^2 \tag{3}$$

where c_p is the fluid specific heat. The energy equation (3) is valid for both an ideal gas (like air) and an incompressible fluid (like water and oil) with zero internal heat generation and negligible compressibility effect (Bejan). For fully developed flow $\boldsymbol{v}=\partial T/\partial x=\mathbf{0}$, the preceding equation reduces to

$$\frac{\partial}{\partial y}\left(k\frac{\partial T}{\partial y}\right)+\mu\left(\frac{\partial u}{\partial y}\right)^2=0 \tag{4}$$

The second term in the energy equation represents the viscous dissipation (heat due to friction).

The boundary conditions at the two plates are

$$y=0,\qquad u=0,\qquad T=T_1 \tag{5}$$

$$y=b,\qquad u=u_2,\qquad T=T_2 \tag{6}$$

where T_1 is the temperature of the lower plate, T_2 is the temperature of the upper plate, u_2 is the velocity of the upper plate, and b is the distance between the plates.

In the present work, the Reynolds number is lower than 1500 (White [1], p. 113) and the flow is laminar.

It should be noted here that the momentum and energy equations are coupled. Each of the four quantities (u, T, μ, and k) depends on the other three. For example, velocity depends on μ directly in the momentum equation; depends on T, because μ is a function of temperature; and depends on k, because k influences the temperature and this influence is transferred to velocity through viscosity μ. Thermal conductivity is a direct function of temperature T; k is influenced by velocity u, because velocity influences the temperature; and k is also influenced by viscosity, because viscosity influences u, u influences T, and T influences k.

Normal air is a transparent, nonparticipating fluid and the boundary conditions in the present problem are prescribed temperature at the plates. This means that the standard equations of motion and energy are valid (Ozisik [13]) and, for that reason, radiation has not been included in our analysis.

In some special cases, Eqs. (1) and (4) may accept analytical solutions. In the present work, we solved these equations directly, without any transformation, using the finite difference method of Patankar [14]. The solution procedure starts with a known distribution of velocity and temperature at the channel entrance ($x=0$) and marches along the plates. These profiles were used only to start the computations and their shape had no influence on the results, which were taken far downstream. At the channel entrance, the temperature and velocity were taken uniformly, with a very small value. At each downstream position, the discretized Eqs. (1) and (4) are solved using the tridiagonal matrix algorithm (TDMA). As x increases, the successive velocity profiles become increasingly similar, and the same result happens with temperature profiles. The solution procedure stops at the point at which the successive velocity and successive temperature profiles become identical (fully developed flow, both hydrodynamically and thermally). The forward step size Δx was 0.01 mm with a total of 500 lateral grid cells. The results are grid-independent. The parabolic solution procedure is a well-known solution method and has been used extensively in the literature. It appeared for the first time in 1970 (Patankar and Spalding [15]) and has been included in classical fluid mechanics textbooks (White , p. 275). In the solution procedure, μ and k have been considered as functions of temperature (White, p. 528).

The dimensionless velocity U, the dimensionless temperature θ, and the dimensionless distance Y are given by the following equations:

$$U=u/u_2 \tag{7}$$

$$\vartheta=(T-T_1)/(T_2-T_1) \tag{8}$$

$$Y=y/b \tag{9}$$

When the plate temperatures are equal ($T_1=T_2$), the dimensionless temperature is defined as

$$\vartheta=T/T_r \tag{10}$$

where T_r is the reference temperature $T_r=(T_1+T_2)/2$. The Nusselt numbers at the two plates are (Shah and London [17]):

$$Nu_1=\frac{b}{T_1-T_b}\frac{\partial T}{\partial y}\bigg|_{y=0} \tag{11}$$

$$Nu_2=\frac{b}{T_2-T_b}\frac{\partial T}{\partial y}\bigg|_{y=b} \tag{12}$$

where the bulk temperature is

$$T_b=\frac{\int_0^b T(y)u(y)\,\mathrm{d}y}{\int_0^b u(y)\,\mathrm{d}y} \tag{13}$$

For a fluid with constant properties, the temperature in the fluid is given by the following equation (analytical solution):

$$T=T_1+(T_2-T_1)\frac{y}{b}+\frac{\mu u_2^2}{2k}\frac{y}{b}\left(1-\frac{y}{b}\right) \tag{14}$$

and the velocity is given by

$$u=(y/b)u_2 \tag{15}$$

Substituting temperature and velocity from Eqs. (14) and (15) into Eq. (13), we have the bulk temperature for a fluid with constant properties:

$$T_b=T_1+\frac{2}{3}(T_2-T_1)+\frac{\mu u_2^2}{12k} \tag{16}$$

From Eq. (14), we can calculate the derivatives $\partial T/\partial y$ at $y=0$ and $y=b$. Taking into account that T_b is also known, we can calculate the Nusselt numbers for a fluid with constant properties:

$$Nu_1=-\frac{6(Brc+2)}{(Brc+8)} \tag{17}$$

$$Nu_2=\frac{6(Brc-2)}{(Brc-4)} \tag{18}$$

where Brc is the classical Brinkman number, defined as

$$Brc=\frac{\mu}{k}\frac{u_2^2}{(T_2-T_1)} \tag{19}$$

In the present work, the preceding Brinkman number is unsuitable, because it becomes infinite when the plate temperatures are equal ($T_1=T_2$). For that reason, we used the following Brinkman number:

$$Br=\frac{\mu_r}{k_r}\frac{u_2^2}{T_r} \tag{20}$$

where μ_r is the reference fluid viscosity and k_r is the reference thermal conductivity, both calculated at the reference temperature, where $T_r = (T_1 + T_2)/2$.

For equal plate temperatures and constant fluid properties, the Nusselt numbers take the values $Nu_1 = -6$ and $Nu_2 = +6$, respectively, whereas for constant fluid properties $T_1 \neq T_2$ and $Brc = 0$, the Nusselt numbers take the values $Nu_1 = -1.5$ and $Nu_2 = 3$, respectively.

III. Results and Discussion

A. Results for Air

For air, the density, dynamic viscosity, and thermal conductivity are given by the following equations for 150 K $\leq T \leq$ 3000 K (Zografos et al. [16]):

$$\rho(\mathrm{kg/m^3}) = 345.57(T - 2.6884)^{-1} \tag{21}$$

$$\mu(\mathrm{Ns/m^2}) = 2.5914 \times 10^{-15}T^3 - 1.4346 \times 10^{-11}T^2 + 5.0523 \times 10^{-8}T + 4.1130 \times 10^{-6} \tag{22}$$

$$k(\mathrm{W/mK}) = 1.5797 \times 10^{-17}T^5 - 9.4600 \times 10^{-14}T^4 + 2.2012 \times 10^{-10}T^3 - 2.3758 \times 10^{-7}T^2 + 1.7082 \times 10^{-4}T - 7.488 \times 10^{-3} \tag{23}$$

In Fig. 1, three temperature profiles are presented for plate temperatures equal to 150 K and Brinkman number 36. We see that the variation of viscosity and thermal conductivity plays an important role in the results. Our temperature profile with constant viscosity and constant thermal conductivity is not included in the figure, because it is completely identical with the analytical solution and this is a validation test to see if our method gives accurate results (if our profile were included, the dashed line curve would disappear).

In Fig. 2, velocity and temperature profiles are presented for different plate temperatures. In the same figure, the results that correspond to a fluid with constant properties (analytical solutions) have been included for comparison (dashed lines). The velocity profiles for a fluid with constant properties are straight lines and independent of the Brinkman number. From Figs. 2a and 2c, we see that air temperature increases as the Brinkman number increases. In Fig. 2a, results are shown for the maximum Brinkman number 256, because for greater Brinkman numbers, the temperature rises above 3000 K and this temperature is outside the range for which Eqs. (22) and (23) are valid. For the same reason, the maximum Brinkman number in Fig. 2c is 16. For the case of equal plate temperatures ($T_1 = T_2$), the real air temperature is lower than that of constant properties in the central region and greater than that of constant properties in the region near the plates. The temperature profiles are symmetric when $T_1 = T_2$ and the same result happens with the corresponding velocity profiles, which take an S form. In Fig. 2c (unequal plate temperatures), it is seen that for low Brinkman numbers, the real air temperature is greater than that of constant properties in the entire cross section, whereas for high Brinkman numbers, the real temperature is greater near the lower plate and smaller near the upper plate in comparison with constant properties temperatures. In Fig. 2d (unequal plate temperatures), it is seen that as the Brinkman number increases, the velocity decreases. The explanation is given by viscosity. We know that air viscosity increases as temperature increases and decreases as temperature decreases. When the Brinkman number increases, the temperature increases, and this increase causes a rise in viscosity. Higher viscosity means lower velocity. (It is known in fluid mechanics that high viscosity hinders velocity and vice versa.)

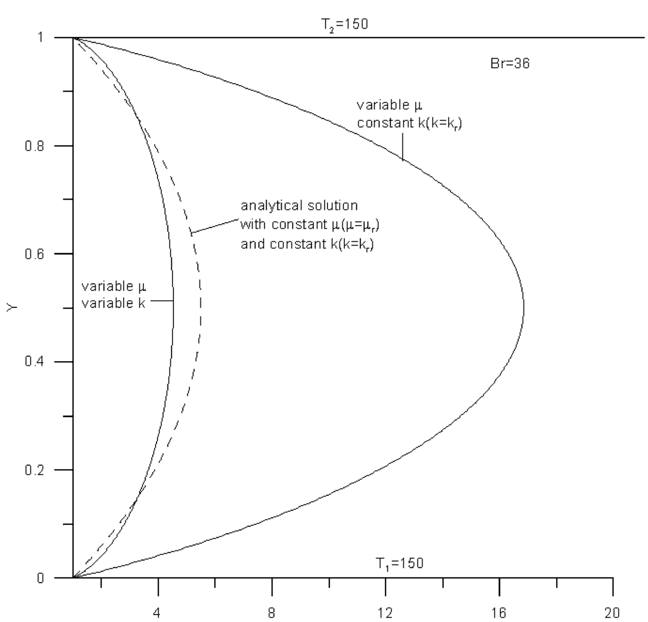

Fig. 1 Influence of air viscosity and thermal conductivity on the temperature profile for *Br* = 36 (solid line, present work; dashed line, analytical solution).

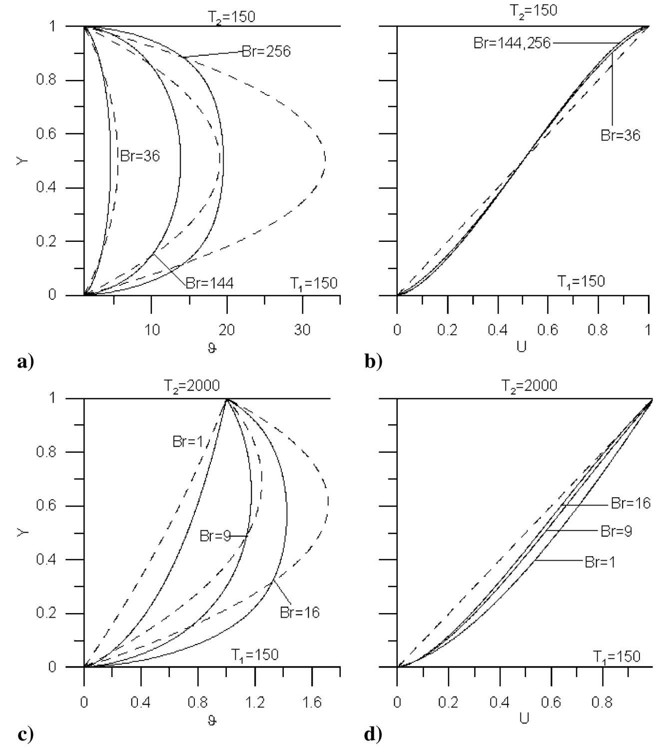

Fig. 2 Temperature and velocity distribution for air [solid line, present work with variable properties; dashed line, solution for a fluid with constant properties (analytical solution)].

The Nusselt numbers calculated by the present method are shown in Table 1. In the same table, the Nusselt numbers for a fluid with constant properties have been also included for comparison (where cp represents constant properties).

For $T_1 = T_2$, the difference between the variable properties' Nusselt numbers and those corresponding to constant properties increases as the Brinkman number increases and vice versa. We see that as $Br \to 0$ the Nusselt numbers tend to −6 and 6. However, for unequal plate temperatures, there is no clear trend, due to complicated interaction between the u, T, μ, and k.

B. Results for Engine Oil

For engine oil, the density, dynamic viscosity, and thermal conductivity are given by the following equations for 273 K $\leq T \leq$

Table 1 Nusselt numbers for air

Br	Nu_1	Nu_1, cp	Difference % $(Nu_1 - Nu_{1cp})/Nu_1$	Nu_2	Nu_2, cp	Difference % $(Nu_2 - Nu_{2cp})/Nu_2$
$T_1 = T_2 = 150$ K						
0.1	−6.04	−6	1	6.04	6	1
1	−6.42	−6	7	6.42	6	7
10	−9.41	−6	36	9.41	6	36
100	−24.77	−6	76	24.77	6	76
256	−40.56	−6	85	40.56	6	85
$T_1 = 150$ K $T_2 = 2000$ K						
0.1	−7.25	−1.53	79	2.07	2.96	43
1	−8.98	−1.80	80	1.86	2.49	34
10	−22.93	−3.39	85	12.48	12.64	1
16	−30.33	−3.92	87	10.72	8.27	23

Table 2 Nusselt numbers for oil

Br	Nu_1	Nu_1, cp	Difference % $(Nu_1 - Nu_{1cp})/Nu_1$	Nu_2	Nu_2, cp	Difference % $(Nu_2 - Nu_{2cp})/Nu_2$
$T_1 = T_2 = 273$ K						
0.1	−5.80	−6	3	5.80	6	3
1	−5.02	−6	20	5.02	6	20
10	−4.36	−6	38	4.36	6	38
100	−4.24	−6	42	4.24	6	42
900	−4.57	−6	31	4.57	6	31
$T_1 = 273$ K $T_2 = 400$ K						
0.1	−1.25	−1.64	31	5.05	2.79	45
1	−1.42	−2.62	85	2.93	−2.89	-
10	−2.10	−4.96	136	13.17	6.53	50
12.25	−2.17	−5.11	135	11.38	6.42	44

Table 3 Nusselt numbers for water

Br	Nu_1	Nu_1, cp	Difference % $(Nu_1 - Nu_{1cp})/Nu_1$	Nu_2	Nu_2, cp	Difference % $(Nu_2 - Nu_{2cp})/Nu_2$
$T_1 = T_2 = 273$ K						
0.1	−5.87	−6	2	5.87	6	2
1	−5.68	−6	6	5.68	6	6
4	−5.64	−6	6	5.64	6	6
9	−5.69	−6	5	5.69	6	5
$T_1 = 273$ K $T_2 = 343$ K						
0.1	−1.67	−1.74	4	2.87	2.63	8
0.25	−1.90	−2.05	8	1.95	−1.85	5
1	−2.59	−3.10	20	−105.41	36.0	-
2.25	−3.20	−3.99	25	9.81	8.03	18

430 K (Zografos et al. [16]):

$$\rho(\mathrm{kg/m^3}) = -0.59212T + 1061.3 \tag{24}$$

$$\mu(\mathrm{Ns/m^2}) = 3.0865\exp[-20.306(T/260 - 1.0605)] \quad 273\ \mathrm{K} \le T < 330\ \mathrm{K} \tag{25}$$

$$\mu(\mathrm{Ns/m^2}) = 0.08254\exp[-13.321(T/330 - 0.99988)] \quad 330\ \mathrm{K} \le T < 380\ \mathrm{K} \tag{26}$$

$$\mu(\mathrm{Ns/m^2}) = 0.01396\exp[-8.6352(T/380 - 0.99994)] \quad 380\ \mathrm{K} \le T \le 430\ \mathrm{K} \tag{27}$$

$$k(\mathrm{W/mK}) = -4.29 \times 10^{-4}T + 0.264 \qquad 273\ \mathrm{K} \le T < 280\ \mathrm{K} \tag{28}$$

$$k(\mathrm{W/mK}) = -2.3810 \times 10^{-6}T^2 + 1.3976 \times 10^{-3}T - 6.6025 \times 10^{-2} \qquad 280\ \mathrm{K} \le T \le 350\ \mathrm{K} \tag{29}$$

$$k(\mathrm{W/mK}) = 0.138 \qquad 350\ \mathrm{K} < T < 360\ \mathrm{K} \tag{30}$$

$$k(\mathrm{W/mK}) = 4.881 \times 10^{-9}T^3 - 5.0844 \times 10^{-6}T^2 + 1.6475 \times 10^{-3}T + 2.4042 \times 10^{-2} \qquad 360\ \mathrm{K} \le T \le 430\ \mathrm{K} \tag{31}$$

The temperature profiles in Fig. 3 correspond to plate temperatures equal to 273 K and Brinkman number 1. Our temperature profile with constant viscosity and constant thermal conductivity is again identical with the analytical solution and is not included in the figure.

Here, we see that the role of thermal conductivity, in contrast to oil, is small.

In Fig. 4, temperature and velocity profiles are shown for different plate temperatures. In some profiles, there are points at which velocity or temperature show a small jump (discontinuity). Taking into account the preceding equations, we see that viscosity and thermal conductivity are given by different functions for $273\ \mathrm{K} \leq T \leq 430\ \mathrm{K}$. These small jumps appear at the temperatures at which viscosity and conductivity functions change their form. For cases where $Br = 100$ and 900 in Fig. 4a and for cases where $Br = 4$ and 12.25 in Fig. 4c, the corresponding constant properties temperature profiles (analytical solutions, dashed lines) are not presented. The reason is that constant properties temperature profiles are very large; these profiles suppress the figure and, thus, the figure quality is very low.

From Figs. 4a and 4c we see that oil temperature increases as the Brinkman number increases, and the real temperatures are lower than those of a fluid with constant properties in the entire cross section. The temperature profiles are symmetric when $T_1 = T_2$ and the same result happens with the corresponding velocity profiles, which are again S-shaped. However, now the S form is opposite from that of air. In Fig. 4d (unequal plate temperatures), it is seen that as the Brinkman number increases, the velocity increases, in contrast to what happens in air. The different behavior of oil velocity profiles in comparison with that of air is caused by oil viscosity that increases as temperature decreases and vice versa. When the Brinkman number increases, the temperature increases, and this increase causes a fall in oil viscosity. Lower viscosity results in bigger velocity. We see also that velocity is very small (almost zero) near the lower (cold) plate. This is due to high oil viscosity near the cold plate.

The conclusions drawn from Table 1 are also valid in Table 2. For $T_1 = T_2$, the difference between the variable properties' Nusselt numbers and those corresponding to constant properties increases as the Brinkman number increases and vice versa. However, for unequal plate temperatures, there is no clear trend, due to complicated interaction between the u, T, μ, and k. It is remarkable that for $T_1 = 273$ K, $T_2 = 400$ K, and $Br = 1$, the Nu_2 numbers have opposite signs.

C. Results for Water

For water, the density, dynamic viscosity, and thermal conductivity are given by the following equations for $273\ \mathrm{K} \leq T \leq 600\ \mathrm{K}$ (Zografos et al. [16]):

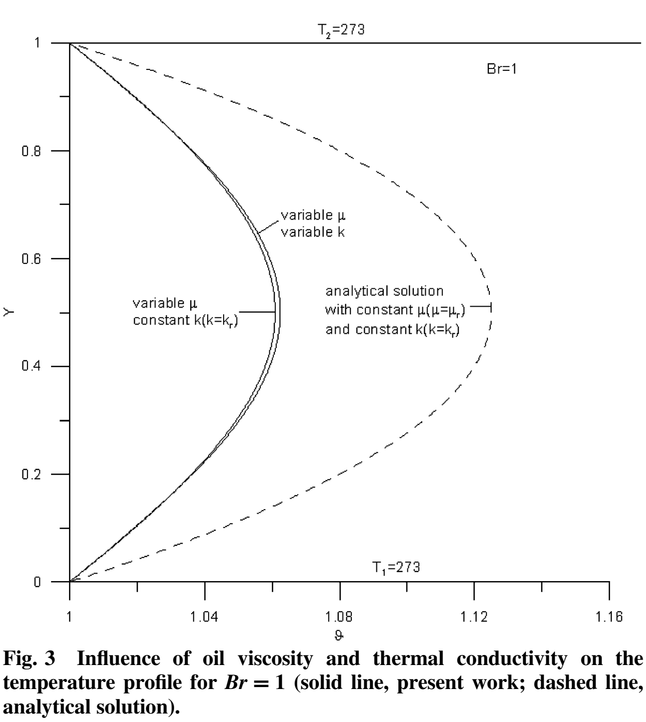

Fig. 3 Influence of oil viscosity and thermal conductivity on the temperature profile for $Br = 1$ (solid line, present work; dashed line, analytical solution).

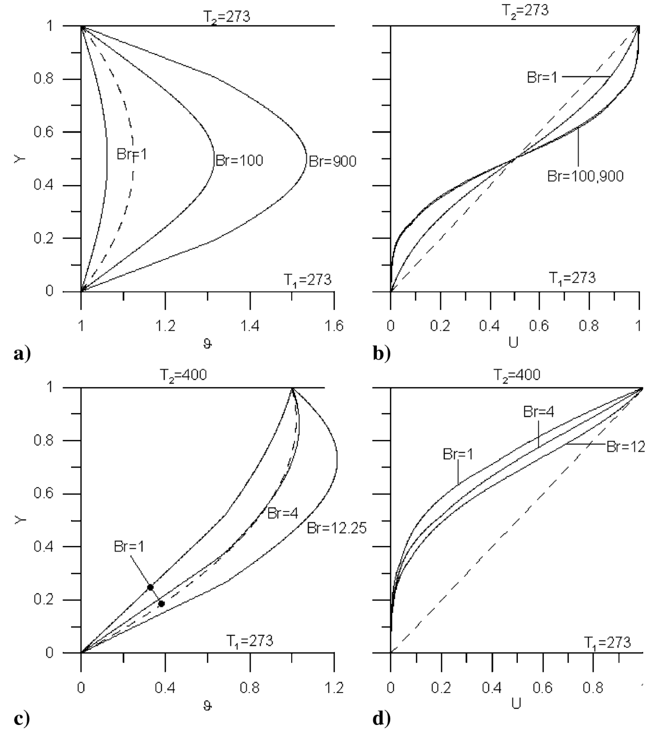

Fig. 4 Temperature and velocity distribution for oil [solid line, present work with variable properties; dashed line, solution for a fluid with constant properties (analytical solution)].

$$\rho(\mathrm{kg/m^3}) = -3.0115 \times 10^{-6} T^3 + 9.6272 \times 10^{-4} T^2 - 0.11052T + 1022.4 \tag{32}$$

$$\mu(\mathrm{Ns/m^2}) = 3.8208 \times 10^{-2} (T - 252.33)^{-1} \tag{33}$$

$$k(\mathrm{W/mK}) = 4.2365 \times 10^{-9} T^3 - 1.1440 \times 10^{-5} T^2 + 7.1959 \times 10^{-3} T - 0.63262 \tag{34}$$

The results are shown in Fig. 5 and Table 3. For cases where $Br = 4$ and 9 in Fig. 5a and for the case where $Br = 2.25$ in Fig. 5c, the corresponding profiles of analytical solutions are not presented, because these profiles suppress the figure. The results in Fig. 5 are qualitatively similar to those for oil, because the variation of viscosity with temperature in the two fluids is similar (viscosity increases as temperature decreases and vice versa). However, the viscosity-temperature relationship in oil is stronger than that of water and, for that reason, the oil results are more vigorous.

Here, we see that the difference between the variable properties' Nusselt numbers and those corresponding to constant properties is small in comparison to the other two fluids. This happens because water viscosity and thermal conductivity are weak functions of temperature. For $T_1 = 273$ K, $T_2 = 343$ K, and $Br = 1$, the Nu_2 numbers again have opposite signs.

IV. Conclusions

The foregoing results are the first complete calculations of the effect of variable viscosity and variable thermal conductivity on the classical plane Couette flow with viscous dissipation and can be summarized as follows:

1) The variation of viscosity with temperature plays an important role in the results. The variation of thermal conductivity is important in air and plays a small role in oil and water.

2) For oil and water, the temperature increases as the Brinkman number increases, whereas in air, different behavior is observed in low and high Brinkman numbers, as well as across the flowfield.

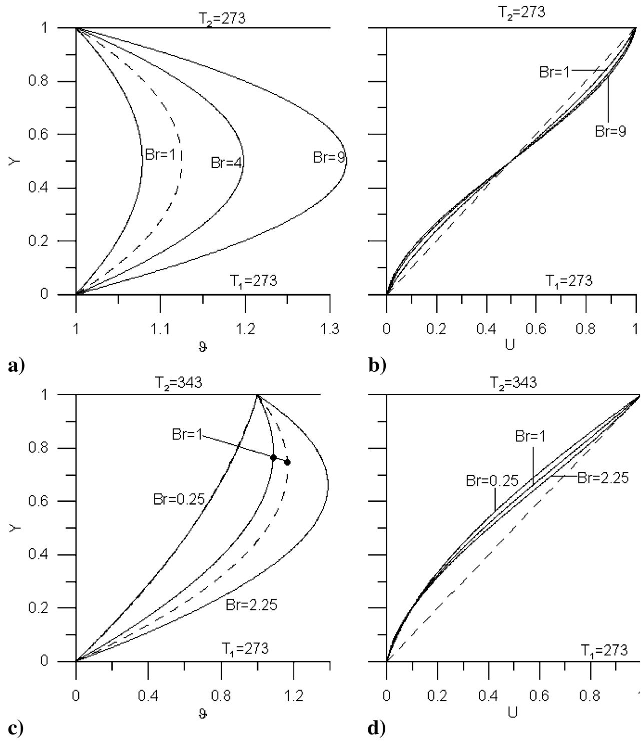

Fig. 5 Temperature and velocity distribution for water [solid line, present work with variable properties; dashed line, solution for a fluid with constant properties (analytical solution)].

3) For equal plate temperatures, the velocity profiles have a symmetric S form that is qualitatively similar in oil and water and reverse in air.

4) For unequal plate temperatures, the velocity increases as the Brinkman number increases for oil and water, whereas the opposite happens in air.

5) For $T_1 = T_2$, the difference between the variable properties' Nusselt numbers and those corresponding to constant properties increases as the Brinkman number increases and vice versa. However, for unequal plate temperatures, there is no clear trend, due to complicated interaction between the u, T, μ, and k. These findings are valid for air and oil.

6) For water, the difference between the variable properties' Nusselt numbers and those corresponding to constant properties is small, because water viscosity and thermal conductivity are weak functions of temperature.

References

[1] White, F., *Viscous Fluid Flow*, 2nd ed., McGraw–Hill, New York, 1991.

[2] Couette, M., "Etudes sur le Frottement des Liquides," *Annales de Chimie et de Physique*, Vol. 21, Ser. 6, 1890, pp. 433–510.

[3] Hestroni, G., Mosyak, A., Pogrebnyak, E., and Yarin, L. P., "Fluid Flow in Microchannels," *International Journal of Heat and Mass Transfer*, Vol. 48, No. 10, 2005, pp. 1982–1998.

[4] Aydin, O., and Kaya, A., "Mixed Convection of a Viscous Dissipating Fluid About a Vertical Flat Plate," *Applied Mathematical Modelling* (to be published).

[5] Barletta, A., Magyari, E., and Keller, B., "Dual Mixed Convection Flows in a Vertical Channel," *International Journal of Heat and Mass Transfer*, Vol. 48, Nos. 23–24, 2005, pp. 4835–4845.

[6] Nield, D. A., Kuznetsov, A. V., and Xiong, M., "Effects of Viscous Dissipation and Flow Work on Forced Convection in a Channel Filled by a Saturated Porous Medium," *Transport in Porous Media*, Vol. 56, No. 3, 2004, pp. 351–367.

[7] Duck, P., Erlebacher, G., and Hussaini, M., "On the Linear Stability of Compressible Plane Couette Flow," *Journal of Fluid Mechanics*, Vol. 258, 1994, pp. 131–165.

[8] Hu, S., and Zhong, X., "Linear Stability of Viscous Supersonic Plane Couette Flow," *Physics of Fluids*, Vol. 10, No. 3, 1998, pp. 709–729.

[9] Makris, N., "Viscous Heating of Fluid Dampers, 1: Small-Amplitude Motions," *Journal of Engineering Mechanics*, Vol. 124, No. 11, 1998, pp. 1210–1216.

[10] Datta, N., Hayes, M. G., Deeth, H. C., and Kelly, A. L., "Significance of Frictional Heating for Effects of High Pressure Homogenisation on Milk," *The Journal of Dairy Research*, Vol. 72, No. 4, 2005, pp. 393–399.

[11] Schlichting, H., *Boundary-Layer Theory*, 6th ed., McGraw–Hill, New York, 1968.

[12] Bejan, A., *Convection Heat Transfer*, Wiley, New York, 1995.

[13] Ozisik, M. N., "Interaction of Radiation with Convection," *Handbook of Single-Phase Convective Heat Transfer*, Wiley, New York, 1987.

[14] Patankar, S. V., *Numerical Heat Transfer and Fluid Flow*, McGraw–Hill, New York, 1980.

[15] Patankar, S. V., and Spalding, D. B., *Heat and Mass Transfer in Boundary Layers*, Intertext, London, 1970.

[16] Zografos, A. I., Martin, W. A., and Sunderland, J. E., "Equations of Properties as a Function of Temperature for Seven Fluids," *Computer Methods in Applied Mechanics and Engineering*, Vol. 61, No. 2, 1987, pp. 177–187.

[17] Shah, R. K., and London, A. L., "Laminar Flow Forced Convection in Ducts," *Supplement 1 to Advances in Heat Transfer*, Academic Press, New York, 1978.

L. Maurice
Associate Editor